SC311
Modeling and Simulation

Lecture 02

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Project (1/2)

Activity	Notes
Project	Self-Study

- **7~10** students per group (Send me at: **ahagag@fci.bu.edu.eg**)
- **Due date: 1-3-2023**
- Follow up the groups in the lectures.
- Final discussion at the end of the semester.

  - Documentation.

  - Discussion.

Project (2/2)

Simulation Software

anylogic

You can find documents and tutorials at this website

https://www.anylogic.com/

Chapter 1: Introduction (1/2)

- General Introduction.
  - ✓ Important Definitions.
  - ✓ When Simulation is the Appropriate Tool.
  - ✓ When Simulation is Not Appropriate.
  - ✓ Advantages and Disadvantages of Simulation.
- Areas of Application.

Chapter 1: Introduction (2/2)

- Systems and System Environment.
- Components of a System.
- Classification of Systems.
- Steps in a Simulation Study.

Systems (1/2)

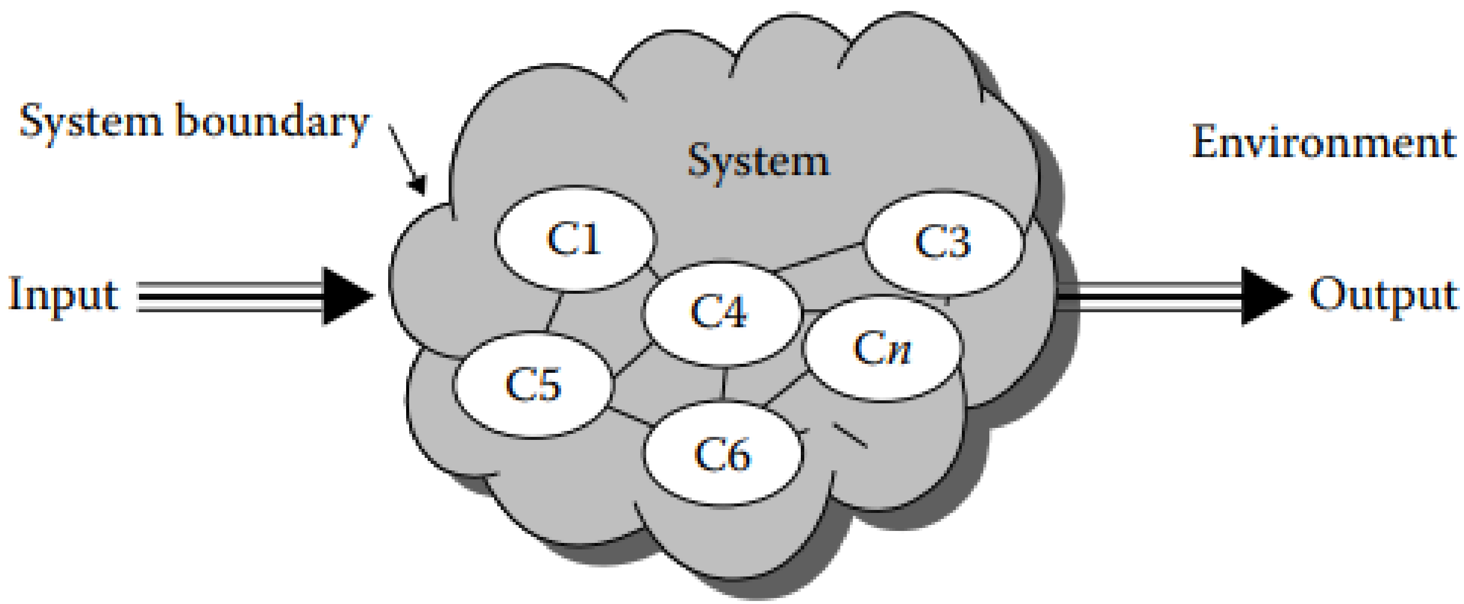
- The term **system** is derived from the Greek word *systema*, which means an organized relationship among functioning units or components. It is used to describe almost any ordered arrangement of ideas or construct.
- A **system** is any set of interrelated components acting together to achieve a common objective.
- A **system** is defined as a group of objects that are joined in some regular interaction or interdependence toward the accomplishment of some purpose.

Systems (2/2)

- The collection of entities that compose a system for one study might be only a subset of another larger system.
  - For example, if one wants to study a banking system to determine the number of tellers needed to provide adequate service for customers who want just to encash or deposit, the system can be defined to be that portion of the bank comprising of the tellers and the customers. Additionally, if, the loan officer and the safety deposit counters are to be included, then the definition of the system must be more inclusive accordingly.

System Environment (1/6)

- A system is characterized by the following attributes:
  - **System boundary.**
  - **System components and their interactions.**
  - **Environment.**



System Environment (2/6)

- A system is often affected by changes occurring outside the system. Such changes are said to occur in the **system environment**.
  - ➢ Factory : Arrival orders

    Effect of supply on demand: relationship between factory output and arrival (activity of system).
  - ➢ Banks : Arrival of customers

System Environment (3/6)

- In modeling systems, it is necessary to decide on the **boundary** between the system (i.e., part of system) and its environment. This decision may depend on the purpose of the study.
- **System component** is a fundamental building block. It is quite easy to find the input–output relations for the system components with the help of some fundamental laws of physics, which is called the mathematical model for components. It may be written in the form of difference or differential equations. They are simple and easily understandable.

System Environment (4/6)

- A living organism is a system. Organisms are open systems: they cannot survive without continuously exchanging matter and energy with their environment. When we separate a living organism from its surrounding, it will die shortly due to lack of oxygen, water, and food.

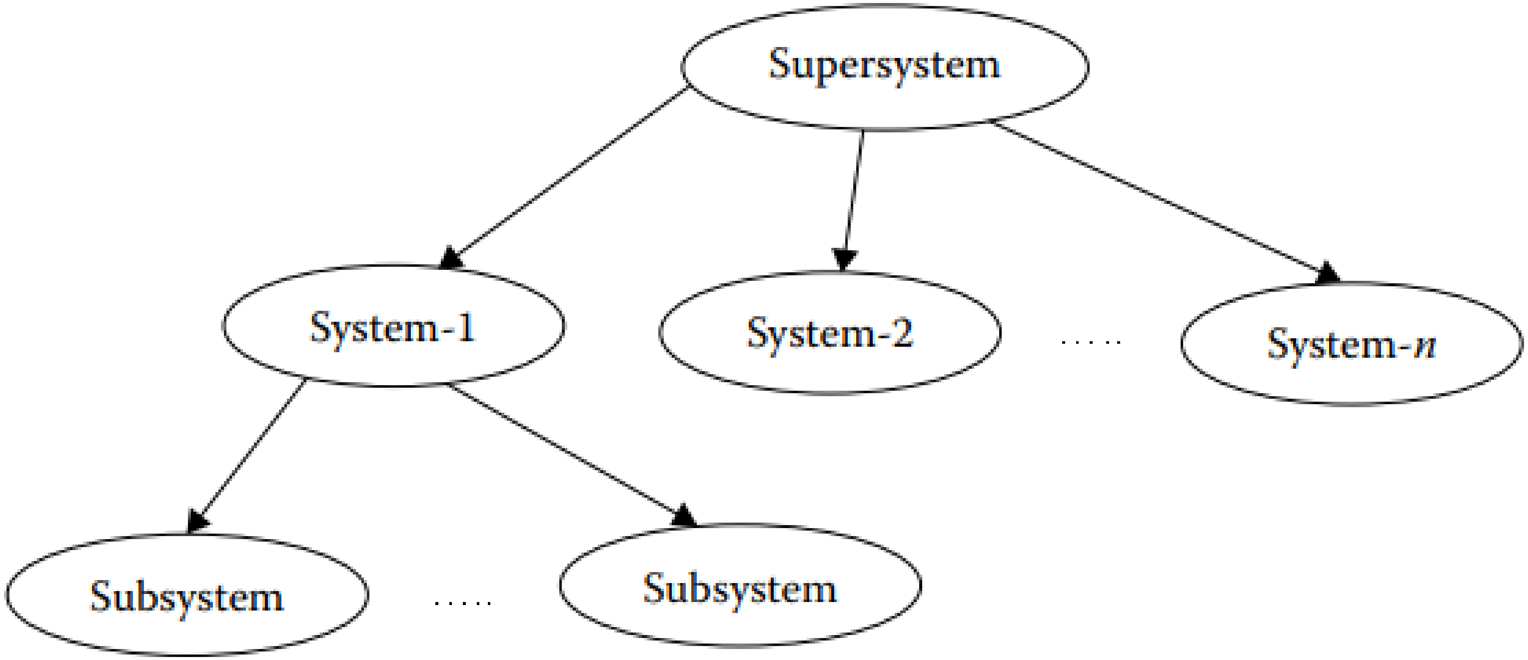
- This interaction has two components: input, that is, what enters the system from outside the boundary, and output, that is, what leaves the system boundary to the **environment**.

System Environment (5/6)

- In order to speak about the inside and the outside of a system, we need to be able to distinguish between the system and its environment, which is in general separated by a **boundary** (for example, living systems, skin is the boundary). The output of a system is generally a direct or indirect result to a given input.

System Environment (6/6)

- Every system consists of subsystem or components at lower levels and supersystems at higher levels.



Components of a System (1/4)

- An **entity** is an object of interest in the system.
  - ➢ Machines in factory.
  - ➢ Customers in Bank.
- An **attribute** is a property of an entity.
  - ➢ Speed, capacity.
  - ➢ Balance in customer accounts.
- An **activity** represents a time period of specified length.
  - ➢ Welding, stamping.
  - ➢ Making deposits.

Components of a System (2/4)

- The **state** of a system is defined to be that collection of variables necessary to describe the system at any time, relative to the objectives of the study.
  - Status of machine (busy, idle, down,…)
  - In the study of a bank, possible state variables are the number of busy tellers, the number of customers waiting in line or being served, and the arrival time of the next customer.

Components of a System (3/4)

- An **event** is defined as an immediate occurrence that might change the state of the system. The term **endogenous** is used to describe activities and events occurring within a system, and the term **exogenous** is used to describe activities and events in the environment that affect the system.
    - In the bank study, the arrival of a customer is an exogenous event, and the completion of service of a customer is an endogenous event.

Components of a System (4/4)

- Examples of entities, attributes, activities, events, and state variables for several systems.

System	*Entities*	*Attributes*	*Activities*	*Events*	*State Variables*
Banking	Customers	Checking-account balance	Making deposits	Arrival; departure	Number of busy tellers; number of customers waiting
Production	Machines	Speed; capacity; breakdown rate	Welding; stamping	Breakdown	Status of machines (busy, idle, or down)
Communications	Messages	Length; destination	Transmitting	Arrival at destination	Number waiting to be transmitted
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

Classification of Systems (1/15)

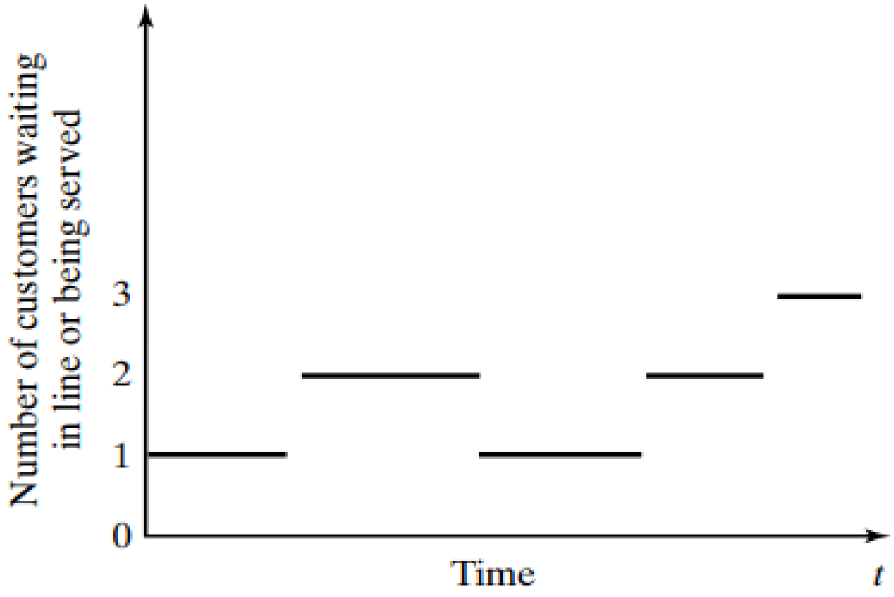
- Systems can be classified based on time frame, type of measurements taken, type of interactions, nature, type of components, etc.

- According to the Time Frame:
  - **Discrete**
  - **Continuous**
  - **Hybrid**

Classification of Systems (2/15)

A **discrete** system:

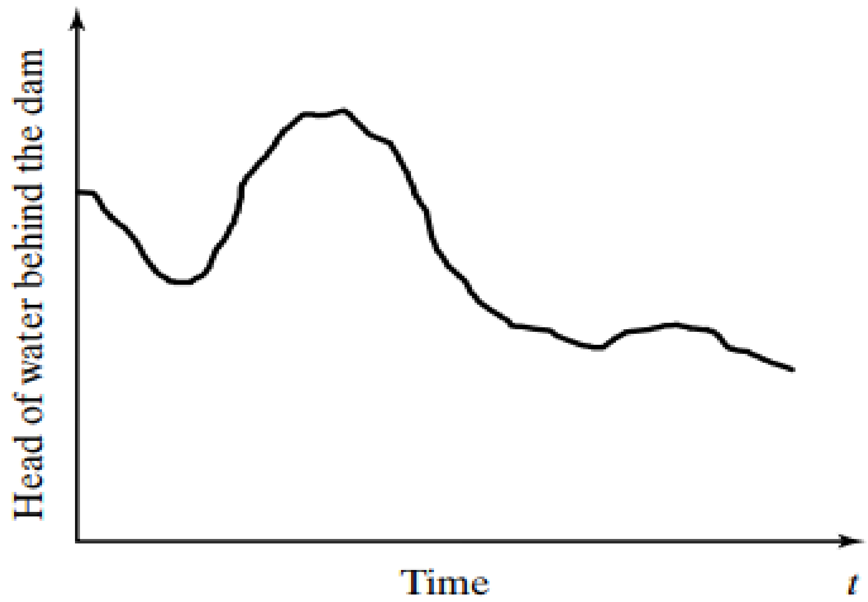
The state variable(s) change only at discrete points in time, for example, queuing systems (bank, telephone network, traffic lights, machine).



Classification of Systems (3/15)

A **continuous** system:

The state variable(s) change continuously over time. An example is the head of water behind a dam, solar system, charging a battery.



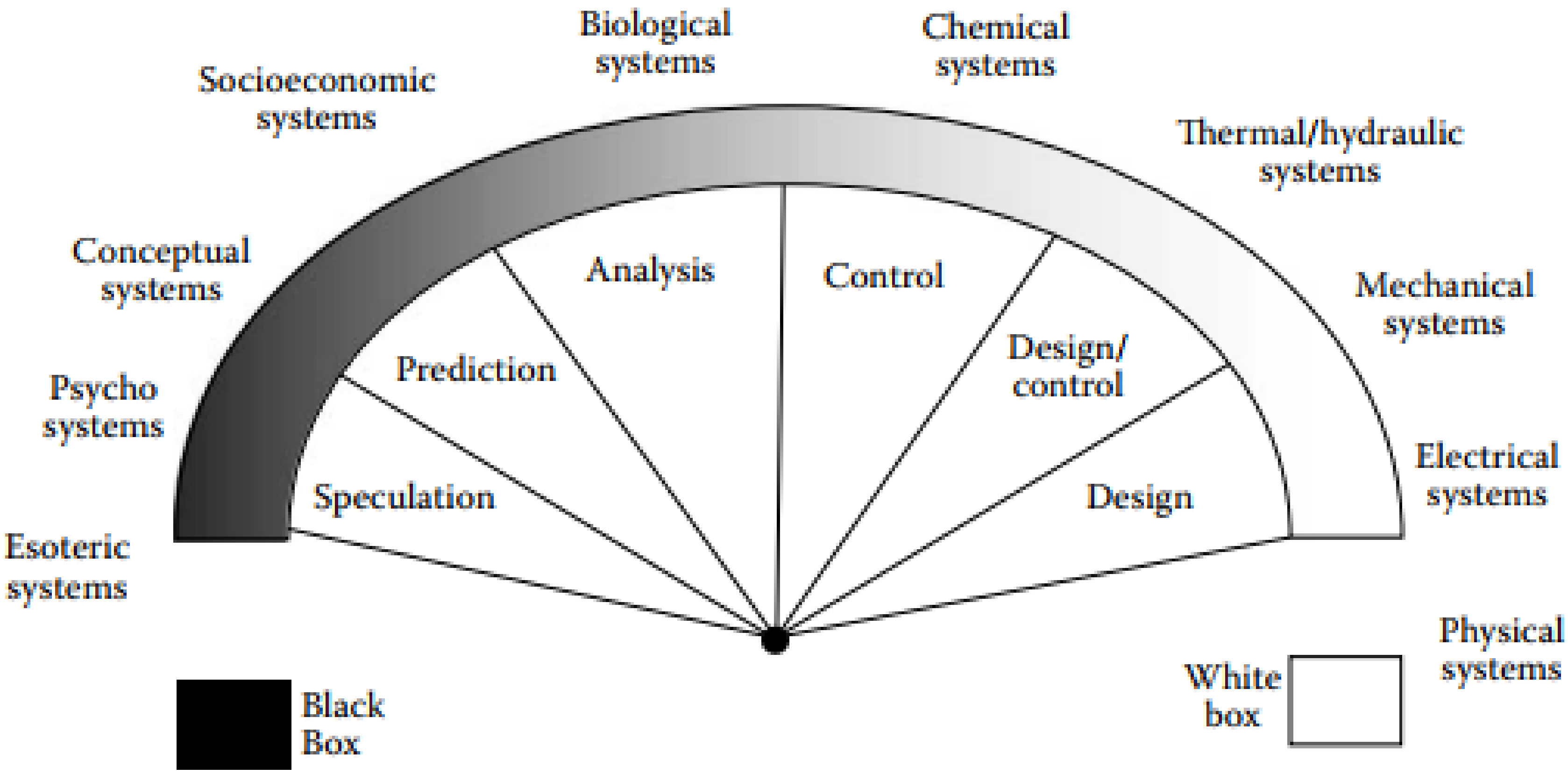
Classification of Systems (4/15)

A **hybrid** system:

A hybrid system is a combination of continuous and discrete dynamic system behavior. A hybrid system has the benefit of encompassing a larger class of systems within its structure, allowing more flexibility in modeling continuous and discrete dynamic phenomena, for example, traffic along a road with traffic lights.

Classification of Systems (5/15)

Classification According to the Complexity of the System:



Classification of Systems (6/15)

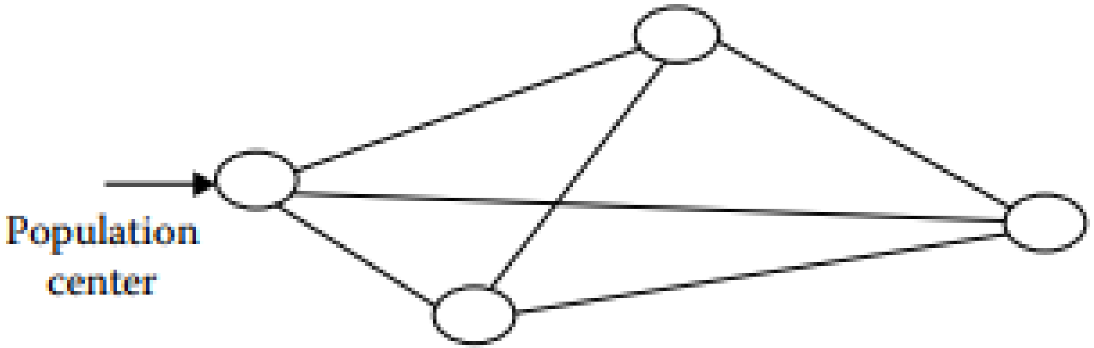
Physical systems:

- Physical systems can be defined as systems whose variables can be measured with physical devices that are quantitative such as electrical systems, mechanical systems, computer systems, hydraulic systems, thermal systems, or a combination of these systems.
- Physical system is a collection of components, in which each component has its own behavior, used for some purpose. These systems are relatively less complex.

Classification of Systems (7/15)

Conceptual systems:

- Conceptual systems are those systems in which all the measurements are conceptual or imaginary and in qualitative form as in psychological systems, social systems, health care systems, and economic systems. These are complex systems.



Transportation system

Classification of Systems (8/15)

Esoteric systems:

- Esoteric systems are the systems in which the **measurements are not possible** with physical measuring devices. The complexity of these systems is of highest order.

Classification of Systems (9/15)

- Systems will be divided into three classes according to the degree of interconnection of events.
  - **Independent**
  - **Cascaded**
  - **Coupled**

Classification of Systems (10/15)

Independent system:

- If the events have no effect upon one another, then the system is classified as independent.

Classification of Systems (11/15)

Cascaded system:

- If the effects of the events are unilateral (that is, part A affects part B, B affects C, C affects D, and not vice versa), the system is classified as cascaded.

Classification of Systems (12/15)

Coupled system:

- If the events mutually affect each other, the system is classified as coupled.

Classification of Systems (13/15)

- Systems can be classified according to <u>the Nature and Type of Components</u>.
  - **Static or dynamic components**
  - **Linear or nonlinear components**
  - **Deterministic or stochastic components**
  - **Continuous-time and discrete-time systems**
  - **Others…**

Classification of Systems (14/15)

Static or dynamic components

- A **static** simulation model, sometimes called a Monte Carlo simulation, represents a system at a particular point in time.
- **Dynamic** simulation models represent systems as they change over time. The simulation of a bank from 9:00A.M. to 4:00P.M. is an example of a dynamic simulation.

Classification of Systems (15/15)

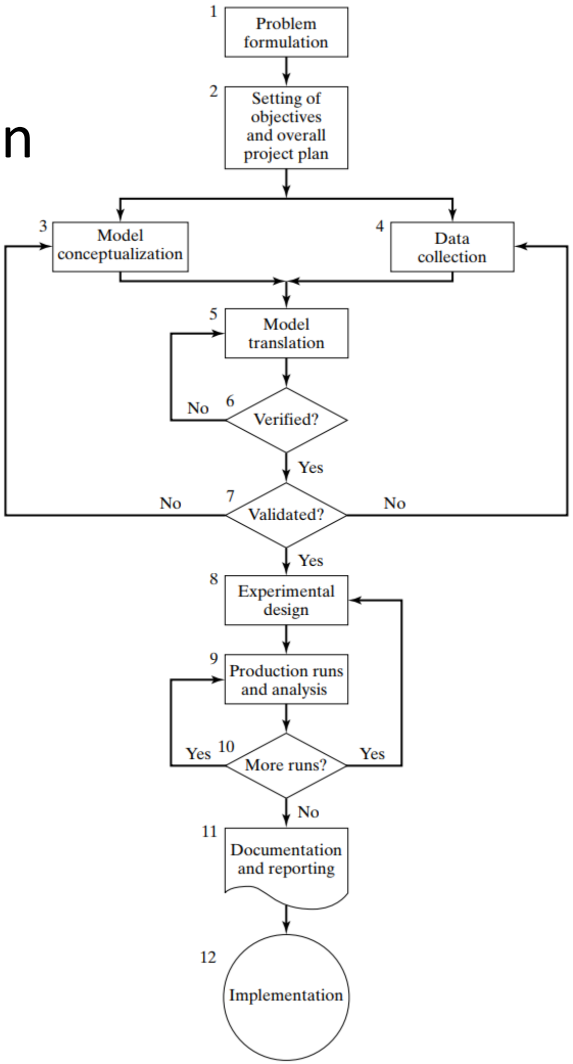
Deterministic or stochastic components

- Simulation models that contain no random variables are classified as **deterministic**. No probabilistic component in the system.

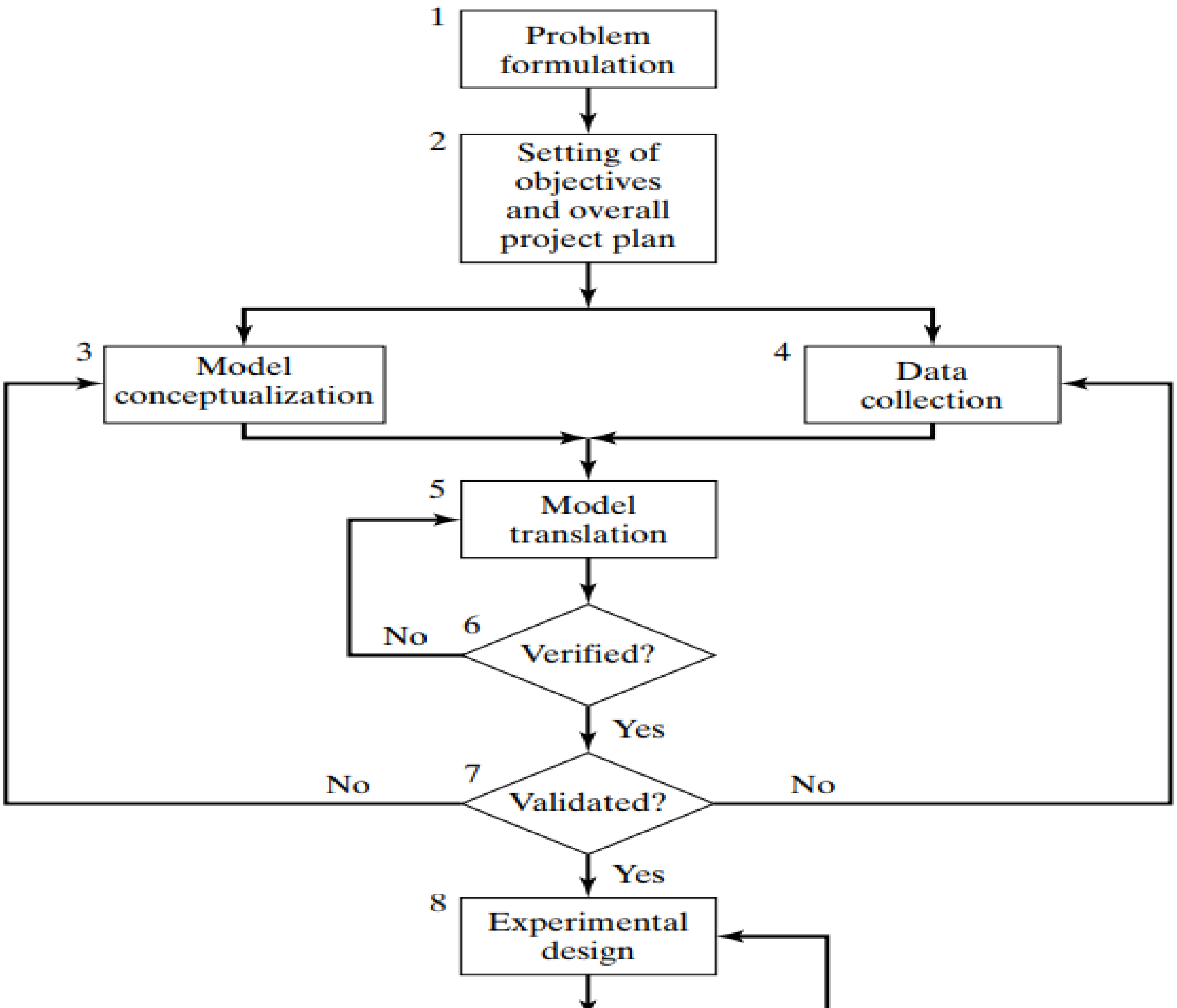
- A **stochastic** simulation model has one or more random variables as inputs. Some components of the system has a probabilistic behavior (Random variable, event probability). Example: Queuing systems.

Steps in a Simulation Study (1/14)

1. Problem formulation
2. Setting of objectives and overall project plan
3. Model conceptualization
4. Data collection
5. Model translation
6. Verified?
7. Validated?
8. Experimental design
9. Production runs and analysis
10. More runs?
11. Documentation and reporting
12. Implementation



Steps in a Simulation Study (2/14)



Steps in a Simulation Study (3/14)

1. Problem formulation

Every study should begin with a statement of the problem. If the statement is provided by the policymakers or those that have the problem, the analyst must ensure that the problem being described is clearly understood. If a problem statement is being developed by the analyst, it is important that the policymakers understand and agree with the formulation.

Steps in a Simulation Study (4/14)

2. Setting of objectives and overall project plan

The objectives indicate the questions to be answered by simulation. At this point, a determination should be made concerning whether simulation is the appropriate methodology for the problem as formulated and the objectives as stated.

Steps in a Simulation Study (5/14)

3. Model conceptualization

It is best to start with a simple model and build toward greater complexity. However, the model complexity need not exceed that required to accomplish the purposes for which the model is intended.

It is advisable to involve the model user in model conceptualization. Involving the model user will both enhance the quality of the resulting model and increase the confidence of the model user in the application of the model.

Steps in a Simulation Study (6/14)

4. Data collection

There is a constant interplay between the construction of the model and the collection of the needed input data. As the complexity of the model changes, the required data elements can also change. Also, since data collection takes such a large portion of the total time required to perform a simulation, it is necessary to begin as early as possible, usually together with the early stages of model building.

Steps in a Simulation Study (7/14)

5. Model translation

Most real-world systems result in models that require a great deal of information storage and computation, so the model must be entered into a computer-recognizable format. We use the term program even though it is possible, in many instances, to accomplish the desired result with little or no actual coding. The modeler must decide whether to program the model in a simulation language or to use special-purpose simulation software.

Steps in a Simulation Study (8/14)

6. Verified?

Did we build the model right?

Verification pertains to the computer program that has been prepared for the simulation model. Is the computer program performing properly? With complex models, it is difficult, if not impossible, to translate a model successfully in its entirety without a good deal of debugging; if the input parameters and logical structure of the model are correctly represented in the computer, verification has been completed.

Steps in a Simulation Study (9/14)

7. Validated?

Did we build the right model?

Validation usually is achieved through the calibration of the model, an iterative process of comparing the model against actual system behavior and using the conflict between the two, and the insights gained, to improve the model. This process is repeated until model accuracy is judged acceptable.

Steps in a Simulation Study (10/14)

8. Experimental design

The alternatives that are to be simulated must be determined. Often, the decision concerning which alternatives to simulate will be a function of runs that have been completed and analyzed. For each system design that is simulated, decisions need to be made concerning the length of the initialization period, the length of simulation runs, and the number of replications to be made of each run.

Steps in a Simulation Study (11/14)

9. Production runs and analysis

Production runs and their subsequent analysis, are used to estimate measures of performance for the system designs that are being simulated.

Steps in a Simulation Study (12/14)

10. More runs?

Given the analysis of runs that have been completed, the analyst determines whether additional runs are needed and what design those additional experiments should follow.

Steps in a Simulation Study (13/14)

11. Documentation and reporting (1/3)

There are two types of documentation: **program** and **progress**.

Program documentation is necessary for numerous reasons.

- If the program is going to be used again by the same or different analysts, it could be necessary to understand how the program operates.
- Also, if the program is to be modified by the same or a different analyst.

Steps in a Simulation Study (13/14)

11. Documentation and reporting (2/3)

Progress reports provide the important, written history of a simulation project.

Project reports give a chronology of work done and decisions made.

Steps in a Simulation Study (13/14)

11. Documentation and reporting (3/3)

"it is better to work with many intermediate milestones than with one absolute deadline."

Possibilities prior to the final report include a model specification, prototype demonstrations, animations, training results, intermediate analyses, program documentation, progress reports, and presentations.

Steps in a Simulation Study (14/14)

12. Implementation

The success of the implementation phase depends on how well the previous eleven steps have been performed.

Probability and Cumulative

Random Experiment

Random (Statistical) Experiment:

- An experiment <with known outcomes> whose outcome cannot be predicted with certainty, before the experiment is run.

Random Experiment

Random (Statistical) Experiment:

- An experiment <with known outcomes> whose outcome cannot be predicted with certainty, before the experiment is run.

The roll of a dice

The toss of (flipping) a coin

Sample Space (1/6)

Sample Space (*S*):

- Set of ALL possible outcomes of a random experiment.
- A sample space is discrete if it consists of a finite or countable infinite set of outcomes.
- A sample space is continuous if it contains an interval (either finite or infinite) of real numbers.

Sample Space (2/6)

Sample Space (*S*):

- Set of ALL possible outcomes of a random experiment.

The roll of a dice

Sample Space (2/6)

Sample Space (S):

- Set of ALL possible outcomes of a random experiment.

The roll of a dice

$$S = \{1,2,3,4,5,6\}$$

Sample Space (2/6)

Sample Space (S):

Discrete

- Set of ALL possible outcomes of a random experiment.

The roll of a dice

$$S = \{1,2,3,4,5,6\}$$

Each outcome in a sample space is called an **element** or a **member** of the sample space, or simply a **sample point**.

Sample Space (3/6)

Sample Space (*S*):

- Set of ALL possible outcomes of a random experiment.

Flipping a coin

Sample Space (3/6)

Sample Space (S):

- Set of ALL possible outcomes of a random experiment.

Flipping a coin

$$S = \{Head, Tail\}$$
$$S = \{H, T\}$$

Sample Space (4/6)

Example1:

Find the sample space for the random experiments (flipping) a coin of two times?

Sample Space (4/6)

Example1:

Find the sample space for the random experiments (flipping) a coin of two times?

Answer:

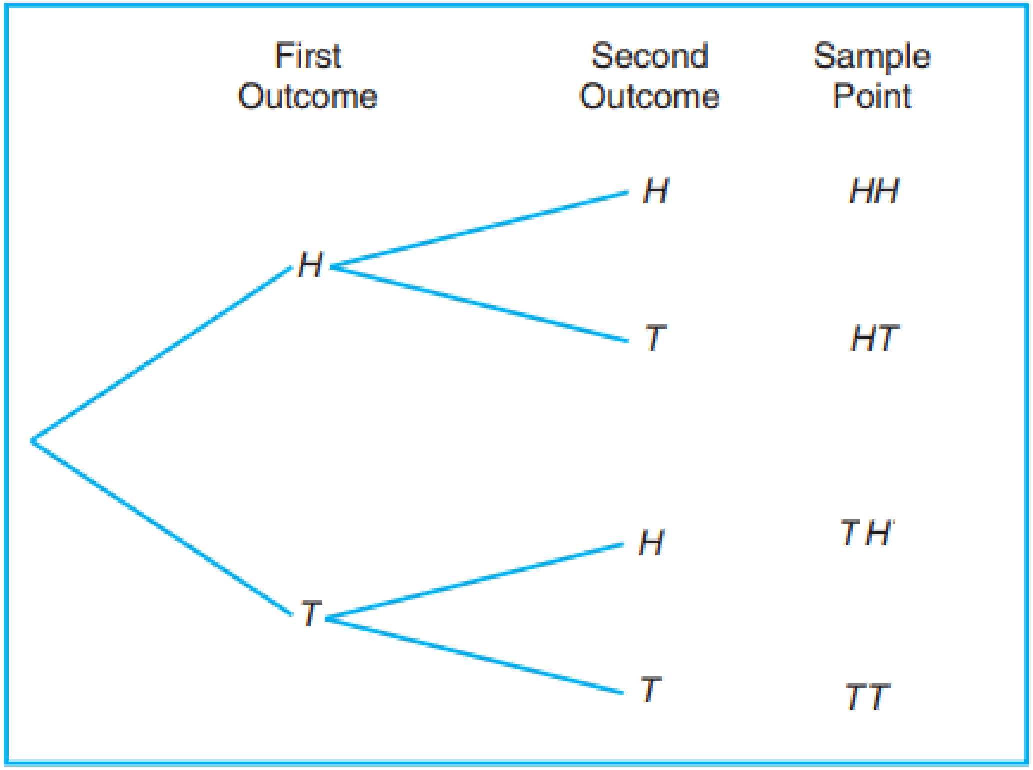
$S = \{HH, HT, TH, TT\}$

Sample Space (5/6)

Tree Diagrams:

Sample spaces can also be described graphically with *tree diagrams*.

$$S = \{HH, HT, TH, TT\}$$



Sample Space (6/6)

Example3:

Continuous

Consider an experiment that selects a cell phone camera and records the recycle time of a flash (the time taken to ready the camera for another flash).

$$\boldsymbol{S} = R^{+} = \{x \mid x > 0\}$$

If it is known that all recycle times are between 1.5 and 5 seconds, the sample space can be

$$\boldsymbol{S} = \{x \mid 1.5 < x < 5\}$$

Events (1/2)

Event (*E*):

- A result of *none* , *one* , or *more* outcomes in the sample space. An event is a subset of the sample space of a random experiment.

Events (2/2)

Event (E):

- A result of *none* , *one* , or *more* outcomes in the sample space. An event is a subset of the sample space of a random experiment.

The roll of a dice

$$S = \{1,2,3,4,5,6\}$$
$$E = \{2,4,6\}$$

Even Numbers

Probability of an Event (1/12)

Probability is used to quantify the likelihood, or chance, that an outcome of a random experiment will occur.

"The chance of rain today is 30%" is a statement that quantifies our feeling about the possibility of rain.

Probability of an Event (2/12)

The likelihood of an outcome is quantified by assigning a number from the interval [0, 1] to the outcome (or a percentage from 0 to 100%).

Higher numbers indicate that the outcome is more likely than lower numbers. A 0 indicates an outcome will not occur. A probability of 1 indicates that an outcome will occur with certainty.

Probability of an Event (3/12)

Equally Likely Outcomes:

Whenever a sample space consists of N possible outcomes that are equally likely, the probability of each outcome is $1/N$.

Probability of an Event (3/12)

Equally Likely Outcomes:

Whenever a sample space consists of N possible outcomes that are equally likely, the probability of each outcome is $1/N$.

Ex. $S = \{1, 2, 3, 4, 5, 6\}$

$$P(1) = \frac{1}{6}, \qquad P(2) = \frac{1}{6}, \qquad \ldots \qquad , \qquad P(6) = \frac{1}{6}$$

Probability of an Event (4/12)

Equally Likely Outcomes:

- S Is a sample space, E is an event
- $E \subseteq S$

$$P(E) = \frac{\#\text{of outcomes in event}(E)}{\text{Total \# of outcomes in sample space}(S)} = \frac{n(E)}{n(S)}$$

$$1 \geq P(E) \geq 0$$

Probability of an Event (5/12)

Example1:

A dice is rolled once. What is the probability of the Event that contains a prime number from the sample space?

Probability of an Event (6/12)

Example1:

A dice is rolled once. What is the probability of the Event that contains a prime number from the sample space?

Solution:

$S = \{1,2,3,4,5,6\}$

$E = \{2,3,5\}$

$$P(E) = \frac{3}{6} = 0.5$$

Probability of an Event (7/12)

Example2:

A coin is tossed twice. What is the probability that at least 1 head occurs?

Probability of an Event (8/12)

Example2:

A coin is tossed twice. What is the probability that at least 1 head occurs?

Solution:

$$S = \{HH, HT, TH, TT\}$$

$$E = \{HH, HT, TH\}$$

$$P(E) = \frac{3}{4}$$

Probability of an Event (9/12)

NOT Equally Likely Outcomes:

A random experiment can result in one of the outcomes $\{a, b, c, d\}$ with probabilities 0.1, 0.3, 0.5, and 0.1, respectively. Let A denote the event $\{a, b\}$, B the event $\{b, c, d\}$, and C the event $\{d\}$. Then,

$$P(A) = 0.1 + 0.3 = 0.4$$

$$P(B) = 0.3 + 0.5 + 0.1 = 0.9$$

$$P(C) = 0.1$$

Probability of an Event (10/12)

Probability of an Event:

For a discrete sample space, the probability of an event E, denoted as $P(E)$, equals the sum of the probabilities of the outcomes in E.

Discrete Random Variables (1/3)

Random Variable

- Is a function that assigns a real number to each outcome in the sample space of random experiment. Denoted by an uppercase letter such as X

A Discrete Random Variable

- Is a random variable with a finite (or countable infinite) range.
- The possible values of X may be listed as $x_1, x_2, \ldots$

Discrete Random Variables (2/3)

Example1

- Flipping a coin of two times. Let X is the number of heads.

Discrete Random Variables (3/3)

Example1

- Flipping a coin of two times. Let X is the number of heads.

Answer:

$$S = \{HH, HT, TH, TT\}$$

2 1 1 0

$$x = 0, 1, 2$$

$$P(0) = \frac{1}{4}, \qquad P(1) = \frac{2}{4}, \qquad P(2) = \frac{1}{4}$$

Probability Mass Fun. (1/5)

Probability Mass Function

For a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$, a **probability mass function** is a function such that

(1) $f(x_i) \geq 0$

(2) $\sum_{i=1}^{n} f(x_i) = 1$

(3) $f(x_i) = P(X = x_i)$

x_i	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{x_4}$	$\boldsymbol{x_5}$
$f(x_i) = P(x_i)$	$\boldsymbol{P(x_1)}$	$\boldsymbol{P(x_2)}$	$\boldsymbol{P(x_3)}$	$\boldsymbol{P(x_4)}$	$\boldsymbol{P(x_5)}$

Probability Mass Fun. (2/5)

Example1

Verify that the function is a probability mass function:

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Probability Mass Fun. (3/5)

Example1

Verify that the function is a probability mass function:

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Answer:

$\sum P(x_i) = 1$, $P(x_i) \geq 0$

Probability Mass Fun. (4/5)

Example2

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Find:

a. $P(X \leq 2)$

b. $P(X > -2)$

c. $P(-1 \leq X \leq 1)$

d. $P(X \leq -1 \text{ or } X = 2)$

Probability Mass Fun. (5/5)

Example2

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Answer:

a. $P(X \leq 2) = 1$

b. $P(X > -2) = \frac{7}{8}$

c. $P(-1 \leq X \leq 1) = \frac{6}{8}$

d. $P(X \leq -1 \text{ or } X = 2) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8}$

Cumulative Distribution (1/9)

The cumulative distribution function (cdf), denoted by $F(x)$, measures the probability that the random variable X assumes a value less than or equal to x, that is,

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

Cumulative Distribution (2/9)

If X is discrete, then

$$F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$$

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Cumulative Distribution (3/9)

If X is discrete, then

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8
$\boldsymbol{F(x) = P(X \leq x)}$	**1/8**	**3/8**	**5/8**	**7/8**	**8/8**

Cumulative Distribution (4/9)

Example1

$$P(X = 0) = 0.6561 \quad P(X = 1) = 0.2916$$
$$P(X = 2) = 0.0486 \quad P(X = 3) = 0.0036$$
$$P(X = 4) = 0.0001$$

x	0	1	2	3	4
$f(x) = P(X = x)$	0.6561	0.2916	0.0486	0.0036	0.0001

Cumulative Distribution (5/9)

Example1

$$P(X=0)=0.6561 \quad P(X=1)=0.2916$$
$$P(X=2)=0.0486 \quad P(X=3)=0.0036$$
$$P(X=4)=0.0001$$

x	0	1	2	3	4
$f(x)=P(X=x)$	0.6561	0.2916	0.0486	0.0036	0.0001
$\boldsymbol{F(x)=P(X\leq x)}$	**0.6561**	**0.9477**	**0.9963**	**0.9999**	**1**

Cumulative Distribution (6/9)

Example1

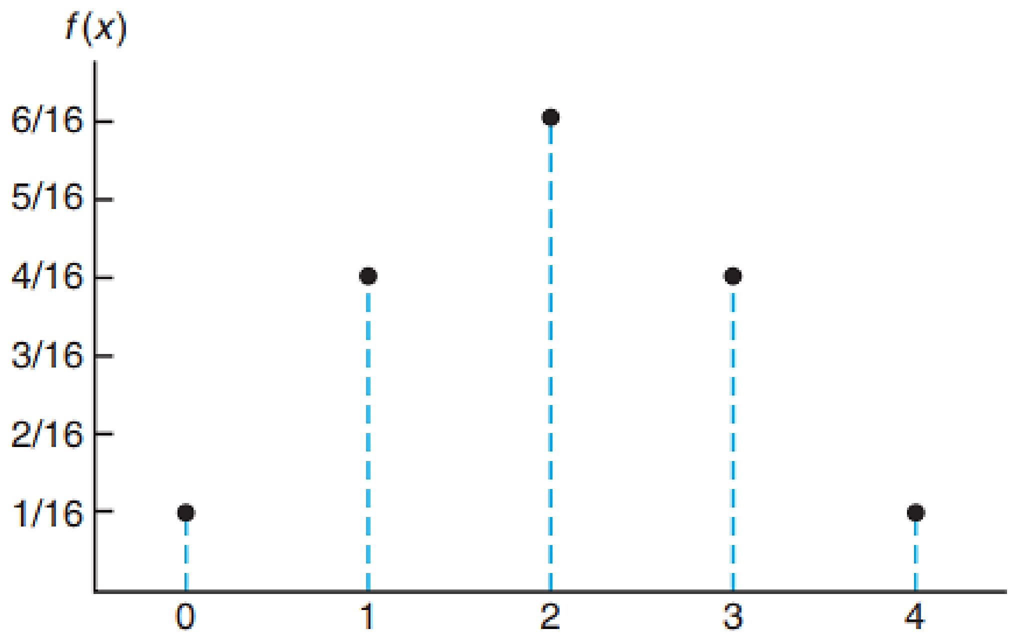
x	0	1	2	3	4
$f(x) = P(X = x)$	0.6561	0.2916	0.0486	0.0036	0.0001
$\boldsymbol{F(x) = P(X \leq x)}$	**0.6561**	**0.9477**	**0.9963**	**0.9999**	**1**

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.6561 & 0 \leq x < 1 \\ 0.9477 & 1 \leq x < 2 \\ 0.9963 & 2 \leq x < 3 \\ 0.9999 & 3 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$$

Cumulative Distribution (7/9)

Example2

x	0	1	2	3	4
$f(x) = P(X = x)$	1/16	4/16	6/16	4/16	1/16



Probability mass function plot.

Cumulative Distribution (8/9)

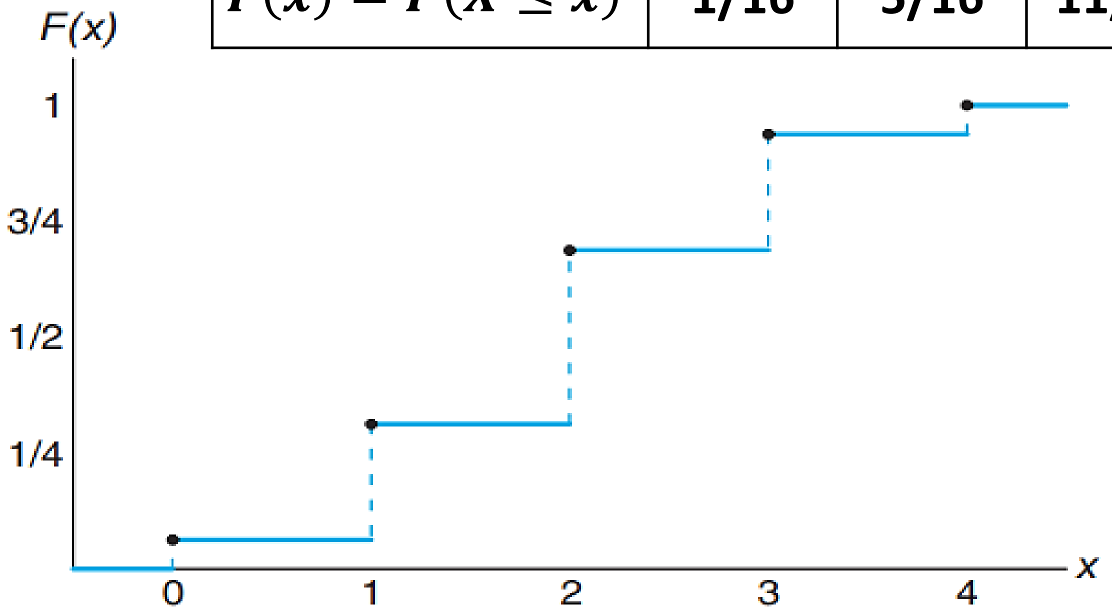
Example2

x	0	1	2	3	4
$f(x) = P(X = x)$	1/16	4/16	6/16	4/16	1/16
$\boldsymbol{F(x) = P(X \leq x)}$	**1/16**	**5/16**	**11/16**	**15/16**	**16/16**

Cumulative Distribution (9/9)

Example2

x	0	1	2	3	4
$f(x) = P(X = x)$	1/16	4/16	6/16	4/16	1/16
$\boldsymbol{F(x) = P(X \leq x)}$	**1/16**	**5/16**	**11/16**	**15/16**	**16/16**



Discrete cumulative distribution function.

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij

Lecture #2: https://www.youtube.com/watch?v=MI3pC54X4KA&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=3

https://www.youtube.com/watch?v=dOsfICOLRIc&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=4

https://www.youtube.com/watch?v=ZT7hZiWVsvE&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=5

https://www.youtube.com/watch?v=_CjF7R8KW-E&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=6

Thank You

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